PROPAGATION AND ATTENUATION OF WAVES GENERATED IN A LAYER BY A PLANE SOURCE

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Piston-in-baffle models are generally used to investigate the excitation of acoustic waves by "opaque" sources (see, e.g., [1-3]). The mathematical problems are simplified considerably by this approach, but such models make it difficult to describe phenomena associated with multiple wave reflections in layered waveguides.

Here we develop a factorization procedure applicable to the solution of initial/boundary-value problems for a compressible fluid, in which a plane "opaque" source is immersed. In contrast with [4, 5], we investigate the space—time structure of the wave fields in the layer on the basis of a numerical-analytic approach. We derive an asymptotic representation of the solution for transient wave radiation and analyze the process of relaxation to steady-state harmonic oscillations.

We consider the excitation of wave fields in a compressible fluid layer by a plane source whose transient oscillations are specified by the law

$$\nabla^2\varphi_\pm = \partial^2\varphi_\pm/\partial t^2,\ -\infty < x_1, x_2 < \infty\ (\mathbf{x} = (x_1, x_2) \in R_2), \tag{1}$$

$$\begin{gathered} p_\pm = -\,\partial\varphi_\pm/\partial t, \quad w_\pm = \partial\varphi_\pm/\partial z, \quad \nabla^2 = \partial^2/\partial x_1^2 + \partial^2/\partial x_2^2 + \partial^2/\partial z^2; \\ z = 1\ \partial\varphi_+/\partial t = 0, \end{gathered} \tag{2}$$

$$\begin{gathered} z = 0 \quad w_+ - w_- = \begin{cases} 0, & \mathbf{x} \notin \Omega, \\ \Delta w, & \mathbf{x} \in \Omega, \end{cases} \quad p_+ - p_- = \begin{cases} 0, & \mathbf{x} \notin \Omega, \\ \Delta p, & \mathbf{x} \in \Omega, \end{cases} \\ z = -H \quad \partial\varphi_-/\partial z = 0; \\ t = 0 \quad \varphi_\pm = 0, \quad \partial\varphi_\pm/\partial t = 0. \end{gathered} \tag{3}$$

Here (x_1, x_2, z) is a Cartesian coordinate system, the domain Ω occupied by the source belongs to the x_1x coordinate plane, ρ and C are the density and sound velocity in the layer, h_+ is the depth of the source, $\varphi_\pm$ is the velocity potential in the layer above (+) and below (−) the plane of the source, and $w_\pm$ and $p_\pm$ are the vertical components of the velocity and the acoustic pressure in the upper and lower sublayers. The computation fulfills the natural physical condition decreasing the resolution at infinity for $x_1^2 + x_2^2 \to \infty$.

A velocity jump Δw is given on the surface of the radiator, where it is expressed in terms of the velocity functions $w_\pm$, which are known at $\mathbf{x} \in \Omega$. The pressure difference Δp and the pressure functions $p_\pm$ at $\mathbf{x} \in \Omega$ are not known.

The dimensionless quantities in Eqs. (1)-(3) are reduced from their original dimensioned counterparts (labeled with an asterisk *) by the relations

$$\begin{gathered} \{x_1, x_2, z\} = \{x_1^*, x_2^*, z^*\}/h_+, \quad t = t^*(C/h_+), \\ p_\pm = p_\pm^*/(\rho C^2), \quad w_\pm = w_\pm^*/C, \quad \varphi_\pm = \varphi_\pm^*/(h_+ C). \end{gathered}$$

The integral equation of the basic problem (1)-(3) is formulated by Fourier integral transformation with respect to the coordinates x_1, x_2 and Laplace transformation with respect to the time t in conjunction with superposition of the solutions of the simpler problems for the upper and lower layers subject to the matching conditions (2) at the interface (z = 0). We have

$$\int\limits_\Omega k_0(\mathbf{x} - \boldsymbol{\alpha}, s)\,\Delta p(\boldsymbol{\alpha}, s)\,d\boldsymbol{\alpha} = f(\mathbf{x}, s), \quad \mathbf{x} \in \Omega, \quad \operatorname{Re} s > s_1 \geqslant 0,$$

$$f(\mathbf{x}, s) = w(\mathbf{x}, s) + \int\limits_\Omega k_1(\mathbf{x} - \boldsymbol{\alpha}, s)\,\Delta w(\boldsymbol{\alpha}, s)\,d\boldsymbol{\alpha},$$

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$$k_m(\mathbf{x}, s) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} K_m(\alpha, s)\, e^{-i\alpha x}\, d\boldsymbol{\alpha}, \quad m = 0, 1; \tag{4}$$

$$K_0(\alpha, s) = \frac{\gamma}{s} \frac{\operatorname{sh} \gamma H \operatorname{ch} \gamma}{\operatorname{ch} \gamma (H+1)}, \quad K_1(\alpha, s) = \frac{\operatorname{sh} \gamma H \operatorname{sh} \gamma}{\operatorname{ch} \gamma (H+1)}, \tag{5}$$
$$w(\mathbf{x}, t) = w_-(\mathbf{x}, t),\ \gamma^2 = \alpha^2 + s^2,\ \boldsymbol{\alpha} = (\alpha_1, \alpha_2),\ \alpha = |\boldsymbol{\alpha}|$$

(s_1 is the abscissa of convergence of the Laplace transform, and α is the modulus of the wave vector). The integral equation is then solved by factorization. The functional equation equivalent to (4) has the form

$$K_0(\alpha, s)\Delta P(\boldsymbol{\alpha}, s) = F(\boldsymbol{\alpha}, s) + \Psi(\boldsymbol{\alpha}, s),$$
$$\boldsymbol{\alpha} \in D,\ \operatorname{Re} s > s_1 \geqslant 0,\ F(\boldsymbol{\alpha}, s) = W(\boldsymbol{\alpha}, s) + K_1(\alpha, s)\Delta W(\boldsymbol{\alpha}, s). \tag{6}$$

Here $\Psi(\alpha, s)$ is the transform of the continuation of the known function $f(x, s)$ into $R_2\backslash\Omega$, and D is the common area of regularity of the functions involved in Eq. (6).

To solve Eq. (6), we investigate the properties of the Green's function K_0. The latter is a single-valued analytic function, whose only singularities are a countable set of zeros $\alpha = \alpha_{nk}(s)$ and poles $\alpha = \eta_n(s)$.

To obtain a single-valued dependence for the dispersion relations, we write them in the form

$$\alpha_{nk}^{\pm} = \pm i\sqrt{s^2 + b_{nk}^2}, \quad b_{n1} = \pi(n-1)/H, \quad b_{n2} = 0{,}5\pi(2n-1),$$
$$\eta_n^{\pm} = \pm i\sqrt{s^2 + a_n^2}, \quad a_n = 0{,}5\pi(2n-1)/(H+1), \quad n = 1, 2, \ldots, \tag{7}$$

where the principal value of the square root is taken as its correct branch, corresponding to an odd dependence of the real values of α_{nk} and η_n for $s = -i\omega$ ($\operatorname{Im}\omega = 0$) on the parameter ω [6].

The singularities of $K_1(\alpha, s)$ can be obtained from the singularities of $K_0(\alpha, s)$ by replacing the set of zeros $\alpha = \alpha_{n2}^{\pm}(s)$ and $\alpha = \alpha_{n3}^{\pm}(s)$ for $b_{n3} = \pi(n-1)$.

The adopted dispersion law (7) fixes $\alpha_{nk}^{\pm}$ and $\eta_n^{\pm}$ in the upper (+) or lower (–) half-planes of the complex plane of α if $\operatorname{Re} s > 0$.

Consequently, for the appropriate choice of perturbing factors w and Δw Eq. (6) is defined in a strip D, which does not have the indicated singularities and contains the entire real axis. Equation (6) can be solved by the method of factorization of functions [7, 8].

It is a well-known fact that the solution of the basic problem can be made unique by imposing conditions on the contour of the radiator such that the energy boundedness requirement is fulfilled. In the planar case this requirement essentially states that the singularity of the solution on the line of demarcation of the boundary conditions ($\chi = 0$) does not exceed an amount of the order of $|\chi|^{-0.5}$ in the limit $|\chi| \to 0$.

We carry out the subsequent analysis in the example of symmetric strip radiators ($\Delta w = 0$, $x \in R_1$).

The factorization method can be used to formulate the functions $\Delta P(\alpha, s)$ and $\Psi(\alpha, s)$ and, accordingly, to obtain integral representations of the wave fields in the entire medium. For example, we obtain expressions for the pressures in the form

$$p(x, z, t) = \frac{1}{4\pi^2 i} \int\limits_{s_0 - i\infty}^{s_0 + i\infty} ds \int\limits_{-\infty}^{\infty} K_0 M \Delta P \exp(-i\alpha x + st)\, d\alpha,$$
$$s_0 > s_1 \geqslant 0, \quad M(\alpha, z, s) = \begin{cases} \dfrac{s}{\gamma} \dfrac{\operatorname{sh} \gamma (1 - z)}{\operatorname{ch} \gamma}, & z \in [0, 1], \\ -\dfrac{s}{\gamma} \dfrac{\operatorname{ch} \gamma (H + z)}{\operatorname{sh} \gamma H}, & z \in [-H, 0) \end{cases} \tag{8}$$

[$\Delta P(\alpha, s)$ depends on the geometry of the domain Ω].

1. For a semiinfinite radiator (Ω: $x \in [0, +\infty)$) the solution of Eq. (6) has the form

$$\Delta P = \frac{1}{K^+}\left\{\frac{F}{K^-}\right\}^+, \quad \Psi = -K^-\left\{\frac{F}{K^-}\right\}^-.$$

Here we have used the following representations of the factorized functions:

$$K^{\pm}(\alpha, s) = \prod_{n=1}^{\infty} \frac{\alpha - \alpha_n^{\mp}(s)}{\alpha - \eta_n^{\mp}(s)}, \quad K_0 = K^+K^-;$$

$$\left\{\frac{F}{K^-}\right\}^{\pm} = \pm\frac{1}{2\pi i}\int\limits_{\Gamma_{\pm}} \frac{F(\xi)\,d\xi}{K^-(\xi)(\xi-\alpha)}, \quad \operatorname{Im}\alpha \gtrless \operatorname{Im}\xi$$

($\Gamma_{\pm}$ denotes infinite contours in D parallel to the real α axis).

2. For a strip radiator of finite width (Ω: $|x| \leq a$) we use the representation $\Psi = \Psi_1 + \Psi_2$ (Ψ_1 corresponds to the continuous of the right-hand side of Eq. (4) for $x > a$, and Ψ_2 has the same significance for $x < -a$).

We investigate the general representation of the solution (8) for a pulsed radiator with a time-harmonic signal carrier:

$$w(x, t) = b(t)d(x), \ |x| \leqslant a, \ t > 0,$$
$$b(t) = t^N \exp(-\varkappa t), \ \varkappa = \varkappa_0 + i\varkappa_1, \ \varkappa_0, \varkappa_1 \geqslant 0, \ N \geqslant 0. \tag{9}$$

We substitute the transforms of the source functions

$$F(\alpha, s) = B(s)D(\alpha), \ B(s) = N!/(s+\varkappa)^{N+1} \tag{10}$$

in Eq. (8) and compute the outer integral, closing the contours in the left half-plane $\operatorname{Re} s < 0$. We obtain

$$p(x, z, t) = \frac{1}{2\pi}\left\{Q + \sum_{k=1,2} g_k\right\},$$
$$Q = \int\limits_{\Gamma_{1,2}} \operatorname{Res}\{K_0 M \Delta P \exp(st), -\varkappa\}\exp(-i\alpha x)\,d\alpha, \tag{11}$$
$$Q \sim O(\exp(-\varkappa_0 t)), \quad \varkappa_0 > 0, \quad N > 0, \quad g_{1,2} = \sum_{n=1}^{\infty} M_n G_n^{1,2} + O\left(t^{-1,5}\right),$$

$$M_n(z) = \begin{cases} \dfrac{\sin[a_n(1-z)]}{a_n \cos a_n}, & z \in [0, 1], \\[2mm] \dfrac{\cos[a_n(H+z)]}{a_n \sin[a_n H]}, & z \in [-H, 0); \end{cases}$$
$$G_n^{1,2}(x, t) = -\int\limits_{\Gamma_{1,2}} H_n^{1,2}(\alpha)\exp(\pm i\chi\varphi_n(\alpha, \beta))\,d\alpha,$$
$$\beta = t/\chi, \quad \chi = |x| - a, \quad t \gg 1, \quad \varphi_n(\alpha, \beta) = \mu_n^-(\alpha)\beta - \alpha,$$
$$\mu_n^-(\alpha) = -i\sigma_n^-(\alpha), \tag{12}$$
$$H_n^{1,2}(\alpha) = \frac{\sigma_n^{\mp} B(\sigma_n^{\mp})}{(\partial\eta_n^{+}/\partial s)\big|_{s=\sigma_n^{\mp}}} R_n^{1,2}(\alpha),$$
$$R_n^{1,2}(\alpha) = \frac{1}{[K_+^{-1}(\pm\alpha, \sigma_n^{\mp}(\alpha))]'} \sum_{m=1}^{\infty} \frac{\left(\Psi_{2,1}^0(\alpha_m^{\mp}) + D(\alpha_m^{\mp})\right)\exp\left(i a \alpha_m^{+}\right)}{[K^+(\alpha_m^-, \sigma_n^-)]'(\alpha_m^+ \pm \alpha)},$$

where the coefficients $\Psi_1{}^0(\alpha_m{}^+)$ and $\Psi_2{}^0(\alpha_m{}^-)$ are determined from the following systems on systems on the assumption that the transforms of the perturbations functions $D(\alpha)$ do not have poles in the upper and lower half-planes of the complex parameter α:

$$\mathbf{A}\boldsymbol{\Psi}_j^0 = \mathbf{B}_j, \quad j = 1, 2, \quad \boldsymbol{\Psi}_1^0 = \{\boldsymbol{\Psi}_1^0(\alpha_k^+)\}_{k=1}^{\infty}, \quad \boldsymbol{\Psi}_2^0 = \{\Psi_2^0(\alpha_k^+)\}_{k=1}^{\infty},$$
$$A_{kl} = \delta_{kl} - \exp\left(ia\left(\alpha_k^+ + \alpha_l^+\right)\right)\frac{K^+(\alpha_k^+)}{[K^+(\alpha_l^-)]'}\sum_{m=1}^{\infty}\frac{K^+(\alpha_m^+)\exp\left(ia\alpha_m^+\right)}{[K^+(\alpha_m^-)]'(\alpha_m^+ + \alpha_k^+)(\alpha_m^+ + \alpha_l^+)},$$

$$B_{jk} = -\exp\left(i a \alpha_k^+\right) K^+\left(\alpha_k^+\right) \sum_{m=1}^{\infty} \frac{Q_{jm} \exp\left(i a \alpha_m^+\right)}{\left[K^+\left(\alpha_m^-\right)\right]' \left(\alpha_m^+ + \alpha_k^+\right)},$$

$$Q_{jm} = D\left(\alpha_m^{\mp}\right) - K^+\left(\alpha_m^+\right) \sum_{n=1}^{\infty} \frac{D\left(\alpha_n^{\pm}\right) \exp\left(i a \alpha_n^+\right)}{\left[K^+\left(\alpha_n^-\right)\right]' \left(\alpha_n^+ + \alpha_m^+\right)}, \quad j = 1, 2. \tag{13}$$

The upper or lower sign is chosen in Eqs. (12) and (13) in accordance with the respective indices 1 and 2; $s = \sigma_n^{\pm}(\alpha)$ are the inverses of the functions $\alpha = \eta_n^{\pm}(s)$, and $\delta_{k\ell}$ is the Kronecker delta symbol. Here the integrands for a harmonic source ($\kappa_0 = 0$) can be singularities on the real axis $\alpha = \eta_n^-(-i\kappa_1)$ (for G_n^1) and $\alpha = \eta_n^+(-i\kappa_1)$ (for G_n^2), which the contours $\Gamma_{1,2}$ bypass via small semicircles in the upper or lower half-plane of α. Outside the neighborhood of these poles $\Gamma_{1,2}$ coincide with the real axis.

3. Poles do not exist in the case of a pulsed source ($\kappa_0 > 0$), and the integration in $G_n^{1,2}$ is carried out along the real axis. We subject the integrals in Eqs. (12) to an asymptotic analysis in the limit $\chi \to \infty, \beta \le$ const. We use the stationary phase method, allowing for the fact that the phase function $\varphi_n(\alpha, \beta)$ has two real stationary points for $\beta > 1$:

$$\alpha_{st}^{\pm} = \pm a_n / \sqrt{\beta^2 - 1}.$$

We find that the acoustic pressures in the domain of disperse interaction are described by the equations

$$p_{1,2}(x, z, t) = -\frac{\beta\left(\beta^2 - 1\right)^{(2N-3)/4}}{\sqrt{2\pi\chi}} \sum_{n=1}^{\infty} a_n^{3/2} M_n(z) \times$$
$$\times \left\{ \frac{R_n^{1,2}\left(\alpha_{st}^{\pm}\right) \exp\left(i a_n \chi \sqrt{\beta^2 - 1} + i\pi/4\right)}{\left(i a_n \beta + \varkappa \sqrt{\beta^2 - 1}\right)^{N+1}} - \right.$$
$$\left. - \frac{R_n^{1,2}\left(\alpha_{st}^{\mp}\right) \exp\left(-i a_n \chi \sqrt{\beta^2 - 1} - i\pi/4\right)}{\left(-i a_n \beta + \varkappa \sqrt{\beta^2 - 1}\right)^{N+1}} \right\} + O\left(\chi^{-\theta}\right), \tag{14}$$
$$\chi \to \infty,\ 0 < \chi < t\ (1 < \beta = \text{const}),\ \varkappa_0 > 0,\ \varkappa_1 \geqslant 0,\ 0{,}75 \leqslant \theta \leqslant 1{,}5$$

[the function index 1 corresponds to a wave propagating to the right of the source ($x > a$), and the index 2 indicates propagation to the left ($x < -a$)]. It follows from Eq. (14) that the amplitude of the main disturbances decays with time as $t^{-0.5}$ in the limit $t \to \infty$ and decays with distance as $\chi^{-0.5}$ in the limit $\chi \to \infty$.

The limiting value of $\beta = t/\chi \to 1$ ($|\alpha_{st}^{\pm}| \to \infty$) corresponds to the leading edge of the wave packet. Here the solution (except in the case of shock formation) behaves as $|t - \chi|^{N-1}$ ($N > 1$); this fact can be deduced from the integrals (12) by replacing the integrands with their asymptotic representations in the limit $\alpha \to \pm\infty$.

4. In the case of a harmonic radiator that has a time-constant amplitude and is actuated from the rest state, it must be assumed in the solution (9)-(13) that $\kappa_0 = 0$, $\kappa_1 > 0$, and $N = 0$. The term Q in Eq. (11) is determined from the residues and characterizes the steady-state part of the solution. In the domain $\chi < C_n t$ $[C_n^{-1} = \partial\eta_n^-(-i\kappa_1)/\partial\kappa_1]$ the integrands in Eq. (12) decay exponentially in the limit $\chi \to \infty$ on the parts of $\Gamma_{1,2}$ that do not coincide with the real axis. The corresponding parts of the integrals $G_n^{1,2}$ admit a power-law estimate $O(\chi^{-1})$ in the limit $\chi \to \infty$. The remaining parts of the contours $\Gamma_{1,2}$ on the real axis represent the asymptotic contribution of $G_n^{1,2}$, which is calculated as in Sec. 3.

In the domain $\chi > C_n t$ we deform the parts of $\Gamma_{1,2}$ near the corresponding poles, reflecting them about the real axis, and then make analogous asymptotic estimates on the new contours. We obtain

$$p_{1,2}(x, z, t) = Q_{1,2}^0(x, z, t) + g_{1,2}^0(x, z, t), \quad x \gtrless \pm a; \tag{15}$$

$$Q_{1,2}^0(x, z, t) = \exp(-i\varkappa_1 t) \sum_{n=1}^{\infty} S_n^{1,2}(-i\varkappa_1) \times$$
$$\times \exp\left(i\chi\eta_n^+(-i\varkappa_1)\right) H(C_n t - \chi), \quad S_n^{1,2}(-i\varkappa_1) = \varkappa_1 M_n(z) R_n^{1,2}\left(\eta_n^{\mp}(-i\varkappa_1)\right), \tag{16}$$
$$H(\chi) = \begin{cases} 1, & \chi > 0, \\ 0, & \chi \leqslant 0, \end{cases} \quad C_n = \left\{\frac{\partial\eta_n^-(-i\varkappa_1)}{\partial\varkappa_1}\right\}^{-1},$$
$$L = [0{,}5(1 + 2\varkappa_1(H + 1)/\pi)],\ \chi = |x| - a,\ \chi \to \infty.$$

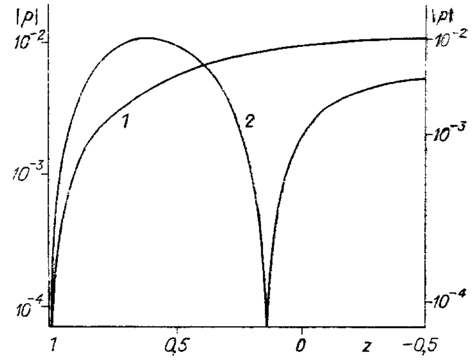

Fig. 1

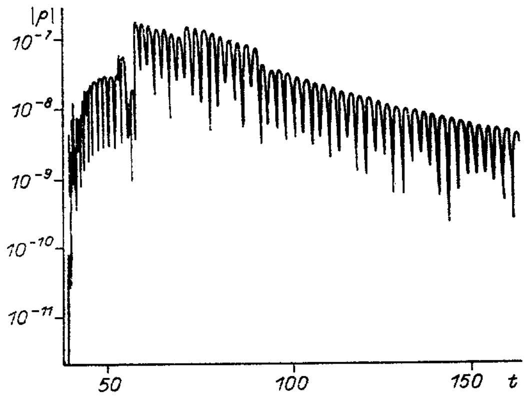

Fig. 2

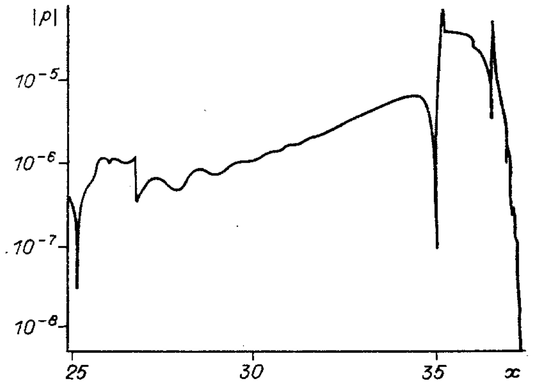

Fig. 3

In this solution $Q_{1,2}{}^0$ denotes steady-state undamped wave contributions, which are superimposed on the transient decaying part $g_{1,2}{}^0$ of the solution, i.e., the response of the medium, which coincides with $p_{1,2}$ (14) if we assume in the latter that $\kappa_0 = 0$, $\kappa_1 > 0$, and $N = 0$. Here L is the number of modes with a real wave number $\eta_n(-i\kappa_1)$, $n = 1, \ldots, L$, generated by a harmonic source at a fixed frequency κ_1.

In Eq. (16) each term of the sum represents the contribution of its corresponding mode to the generated wave packet. Clearly, the velocity of propagation of the leading edge of the corresponding mode coincides with its group velocity. The velocity of the modes decreases monotonically with increasing mode order at a fixed frequency, i.e., $C_n > C_{n+1}$ for all $n = 1, \ldots, L - 1$. This assertion follows from Eq. (7).

Obviously, the solution of the harmonic problem discussed in Sec. 4 has resonances; this fact is inferred from the representation of the wave field (16) for $|x| > a$, where the n-th amplitude of the mode expansion of the field becomes infinite at a fixed resonance frequency $\kappa_n{}^* = a_n$, $n = 1, 2, \ldots$. In the domain outside the source these resonances coincide with the resonances of the problem for a fluid layer with an immersed point source of disturbances. In our problem the presence of an "opaque" strip source of finite width changes the values of the resonances in the domain $|x| < a$ above ($z \in (0, 1]$) and below ($z \in [-H, 0)$) the source. We infer from the general representation of the pressure field (11) that the values $\kappa_n{}^* = b_{n2}$ are critical in the first case, and $\kappa_n{}^* = b_{n1}$ are critical in the second case. It has been shown [8] that the amplitude function grows as $t^{0.5}$ in the limit $t \to \infty$ at resonances of the layer in the planar problem, in contrast with its logarithmic growth at the critical values in the axisymmetric problem [9].

We now give some numerical results obtained according to Eqs. (14)-(16) for a pulsed source of the form (9) [$d = \exp(i\eta x)$, $\eta = 117.8097$] with linear dimension $a = 0.0133$ and thickness of the waveguide $(1 + H)$ $H = 0.3333$. Here we use an appropriate factorization of the Green's function $K_0(\alpha, s)$ in the class of rational functions [8].

Figure 1 shows typical depth distributions of the pressure amplitude in the zone of disperse wave interaction at time $t = 37.3333$ for $\beta = t/\chi = 1.012$ (curve 1) and 1.501 (curve 2) in the case of a radiator without a harmonic carrier ($\kappa_1 = 0$) for a signal with rise-time and decay-time parameters $N = 5$ and $\kappa_0 = 6.4286$ (respectively). The first curve corresponds to the left grid, and the second curve to the right grid. The structure of the distributions depends significantly on the number of modes contributing to the solution.

The nature of the time distribution of the pressure amplitude (beginning with the arrival time of the leading edge) for the indicated waveguide at the point with coordinates $\chi = 37.3333$, $z = 0.1111$ is shown in Fig. 2. The curves correspond to a source with parameters $\kappa_0 = 0.9643$, $\kappa_1 = 0$, and $N = 10$.

Figure 3 illustrates the dependence of the pressure amplitude on the horizontal coordinate x for $z = 0.1111$ at time $t = 37.3333$ in the case of a pulsed source with a carrier $\kappa_0 = 2.3559$, $\kappa_1 = 3.2143$, $N = 5$.

The numerical experiment shows that the domain near the leading edge of the wave packet is determined by a finite number of the fastest first modes with allowance for their multiple reflections. An increasing number of modes contributes to the subsequent structure of the domain, but the influence of their multiple reflections diminishes.

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